The effective action and the triple Pomeron vertex

Martin Hentschinski [a] [b*]

[a] Instituto de Física Teórica UAM/CSIC,
Universidad Autónoma de Madrid, 28049 Madrid, Spain

[b] II. Institute for Theoretical Physics, Hamburg University,
Luruper Chaussee 149, 22761 Hamburg, Germany

We study integrations over light-cone momenta in the gauge invariant effective action of high energy QCD. A regularization mechanism which allows for the evaluation of the longitudinal integrations is presented. After a rederivation of the reggeized gluon and the BFKL-equation from the effective action, we study the $1-3$ and $2-4$ reggeized gluon transition vertex of QCD Reggeon field theory and discuss their connection with the usual triple Pomeron vertex of perturbative QCD.

1. Introduction

In 1995 an effective action [1] for QCD scattering processes at high center of mass energies $\sqrt{s}$ has been proposed by L.N. Lipatov which describes the interaction of fields of reggeized gluons ($A_\pm = -it^a A_\pm^a$) with quark (ψ) and gluon ($v_\mu = -it^a v_\mu^a$) fields, local in rapidity. The effective action reads

$$S_{\text{eff}} = \int d^4x \big(\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\pm, A_\pm)\big), \quad (1)$$

where $\mathcal{L}_{\text{QCD}}$ is the usual QCD-Lagrangian and

$$\begin{aligned} \mathcal{L}_{\text{ind}}(v_\pm, A_\pm) \;&=\; \text{tr}[(A_-(v) - A_-)\partial^2 A_+] \\ &\quad + \text{tr}[(A_+(v) - A_+)\partial^2 A_-]\big) \quad (2) \end{aligned}$$

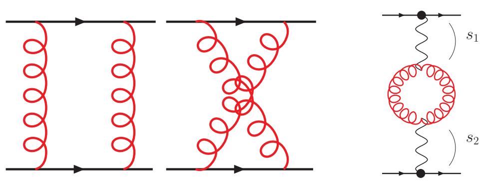
is the *induced* term with

$$\begin{aligned} A_\pm(v) &= v_\pm D_\pm^{-1}\partial_\pm = \\ &= v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \ldots \quad (3) \end{aligned}$$

Light-cone components are defined by $k^\pm \equiv n^\pm \cdot k$ where $n^\pm$ are the light cone directions associated with the scattering particles. The reggeized gluon fields $A_\pm$ are bare reggeized gluons with trajectory $j(t) = 1$, while reggeization occurs as a higher loop correction. The fields $A_\pm$ have the special property to be invariant under local gauge transformations, even though they transform globally in the adjoint representation of $SU(N_c)$. The effective action allows then to factorize high energy QCD amplitudes into gauge invariant pieces which themselves are localized in rapidity. In particular, the interaction between particles and reggeized gluon fields, is by definition restricted to a small rapidity interval $\Delta Y < \eta$, while all non-local interaction, which extends over rapidity intervals $\Delta Y > \eta$, is mediated by reggeized gluons.

Due to these properties the effective action promises to be a suitable tool to study both higher order corrections to the BFKL-equation and unitarization of the BFKL-Pomeron. However, due the factorization of high energy QCD amplitudes into pieces local in rapidity, $Y = \ln(k^+/k^-)/2$, loop integrals over (longitudinal) light-cone momenta take a special role in the effective action. In particular, longitudinal integrals require additional regularization as otherwise the locality principle of the effective action is violated. Transverse integrals on the other hand may be treated by conventional methods. Furthermore, there exists sometimes the danger of overcounting certain parts of the underlying QCD

*This work has been carried out in collaboration with J. Bartels and L.N. Lipatov. Support by the Graduiertenkolleg “Zukünftige Entwicklungen in der Teilchenphysics” and DESY is acknowlegded.



(a) (b)

Figure 1. (a) QCD 1-loop diagrams that yield the leading logarithmic contribution. Their leading $\mathbf{8_A}$-sector is in the effective action shifted to the loop-correction of the reggeized gluon (b).

amplitude within the effective action which can then lead to the occurrence of divergent integrals. In Sec.2 we demonstrate how these points can be addressed in the context of quark-quark scattering within the Leading Logarithmic Approximation (LLA). In Sec.3 we apply the obtained rules to the derivation of reggeized gluon transition vertices and compare them with previous results.

2. The reggeized gluon and the BFKL-Pomeron in the effective action

The Lagrangian of the effective action, Eq. (1), consists apart from the usual QCD Lagrangian of the induced term, Eq. (2). This additional term leads in the effective action to a redistribution of the antisymmetric color octet sector, $\mathbf{8}_A$, of the underlying QCD Feynman diagrams. A 1-loop example for such a re-distribution is shown in Fig. 1. There, in the effective theory diagram Fig. 1.b, the coupling of the reggeized gluon to the gluon loop takes place by the induced vertex $-gv_\pm \frac{1}{\partial_\pm} v_\pm \partial^2 A_\mp$ which arises from Eq. (3). The diagram Fig. 1.b is of particular importance in the effective action, as it yields the 1-loop correction to the trajectory of the reggeized gluon. In order to evaluate the resulting (longitudinal) loop-integrations of Fig. 1.b, it is necessary to take into account that the exchange of the reggeized gluons demands the squared center-of-mass energies s_1 and s_2 of the regarding sub-amplitudes to be large, *i.e.* a corresponding lower bounds needs to be imposed. A simple lower cut-off however misses the imaginary part of Fig. 1.b. It is therefore necessary to use a more elaborated method of regularization. A suitable choice turns out to be the following Mellin-integral

$$\int_{0-i\infty}^{0+i\infty} \frac{d\omega}{4\pi i} \frac{1}{\omega+\nu} \left[\left(\frac{-s_1 - i\epsilon}{\Lambda} \right)^\omega + \left(\frac{s_1 - i\epsilon}{\Lambda} \right)^\omega \right]$$
$$= \theta(|s_1/\Lambda - 1|) \tag{4}$$

where we take the parameter $\nu > 0$ in the limit $\nu \to 0$. In the above expression, $-\nu$ has the interpretation of an infinitesimal small Regge trajectory, while the integration variable ω takes the role of complex angular momentum. This kind of regularization has the great advantage that it allows to take into account in a rather straight forward way phases in s_1 (and therefore also imaginary parts). In particular, negative signature of the reggeized gluon is made explicit. Making use of Eq. (4), the diagram Fig. 1.b can be evaluated and we obtain within the LLA the well-known result for the reggeized gluon at 1-loop:

$$\mathcal{M}_{\text{Fig.1.}b}(s,t) = \mathcal{M}_{\text{tree}}(s,t)\beta(t)\frac{\ln(-s)+\ln s}{2}. \tag{5}$$

Here $\mathcal{M}_{\text{tree}}$ is the quark-quark scattering amplitude at tree-level and $\beta(t)$ is the well-known 1-loop correction to the gluon trajectory. Resummation of diagrams like Fig. 1.b, with an arbitrary number of gluon loops inserted, yields then within the LLA the all order reggeized gluon with negative signature. The leading contribution of the t-channel exchange with positive signature arises on the other hand from the exchange of two reggeized gluons. Typical diagrams with two reggeized gluon exchange that arise from the effective action at 1-loop are shown in Fig. 2. Taking a closer look on the resulting expressions and comparing them with the underlying QCD graphs, Fig. 1.a, it turns out that they all contain a term which occurs already in the diagram Fig. 1.b. Moreover, due to the simplified structure of the reggeized gluon propagator in the diagrams Fig. 2, this term leads for every individual diagram to a divergence in the longitudinal

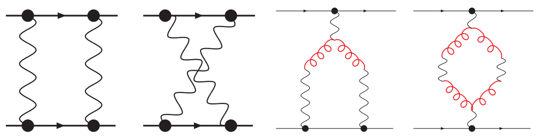


Figure 2. Typical diagrams with exchange of two reggeized gluons in the effective action.

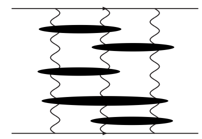


Figure 3. BKP-state of three reggeized gluons with the number of reggeized gluons in the t-channel conserved.

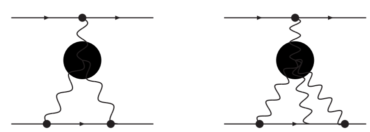


Figure 4. Transition of 1 to 2 and 1 to 3 reggeized gluons inside the elastic quark-quark scattering amplitude.

part of the loop integral. While in the case of two reggeized gluon exchange this problem can be cured by a principal value prescription (see for instance [2]), such a procedure turns out to fail if states with more than two reggeized gluon are considered. A suitable way to deal with this problem is therefore to remove the overcounted terms completely from the regarding diagrams. For general diagrams that contain the exchange of n reggeized gluons, this can be achieved by supplementing the Lagrangian of the effective action by a term

$$\mathcal{L}_{\text{supp}}(A_+, A_-) = -2\mathcal{L}_{\text{ind}}(A_\pm, A_\pm). \tag{6}$$

Note that adding such a term is not in conflict with the original derivation of the effective action. With this term we obtain for the exchange of two reggeized gluons the following result

$$\mathcal{M} = 2\pi i|s| A^{a_1a_2}_{(2,0)} \otimes_{12} A^{a_1a_2}_{(2,0)}, \tag{7}$$

which carries explicitly positive signature. There, the two (reggeized) gluon impact factor $A_{(2,0)}$, is given by

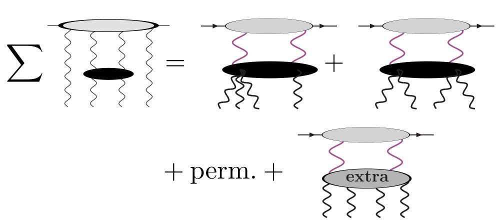
$$A^{a_1a_2}_{(2,0)} = -g^2\frac{1}{2}\left(\frac{1}{N_c}\delta^{a_1a_2} + d^{a_1a_2c}t^c_{AA'}\right), \tag{8}$$

while $\otimes_{12} = \int \frac{d^2\mathbf{k}}{(2\pi)^3\mathbf{k}_1^2\mathbf{k}_2^2}$. The state of two reggeized gluons couples therefore in the color singlet, $\mathbf{1}$, and the symmetric color octet, $\mathbf{8_S}$, to the quark, while the $\mathbf{8_A}$-sector decouples and is contained in the single reggeized gluon exchange. Higher order corrections within the LLA include apart from loop corrections to the individual reggeized gluons, Eq. (5), also the interaction between the reggeized gluons, which yields the (real part) of the BFKL-kernel. Resumming both types of corrections within the LLA, the famous BFKL-equation is re-obtained. The color singlet sector then yields the BFKL-Pomeron, while in the $\mathbf{8_S}$ sector further reggeization occurs. Unlike the $\mathbf{8_A}$-reggeized gluon, the $\mathbf{8_S}$-Reggeon is not a fundamental degree of freedom in the effective action, but arises as a state of two reggeized gluons.

3. Vertices in QCD Reggeon field theory

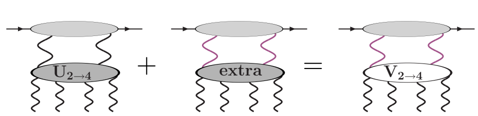
Beyond the state of two reggeized gluons, one is at first lead to so-called BKP-states *i.e.* states of n reggeized gluons, with the number of reggeized gluons in the t-channel conserved. Within the LLA the reggeized gluons interact pairwise by the BFKL-Kernel, see Fig. 3, and the whole system is known to be integrable in the large N_c limit. In a next step one should further take into account transition vertices which change the number of the reggeized gluons in the t-channel. With the rules for longitudinal integrations introduced above, the effective action can be used to derive these vertices. As pointed out by Gri-

bov in [3], signature is conserved within the elastic scattering amplitude. Generally, odd number state of reggeized gluons carry negative signature and even number states of reggeized gluons positive signature. The transition of one to two reggeized gluons, Fig. 4, contradicts therefore signature conservation. At 1-loop this requirement turns out to be automatically fulfilled in the effective action, as the one-to-two transition vanishes inside the elastic scattering amplitude. The transition from one-to-three reggeized gluons is on the other hand allowed by signature conservation and a non-zero transition vertex $U_{1\to 3}$ can be derived from the effective action. A similar result holds for the transition from two-to-three an two-to-four reggeized gluons. While the former vanishes if inserted in the elastic scattering amplitude, the latter is allowed by signature conservation and yields a non-zero vertex[2], $U_{2\to 4}$. The transition vertex $U_{2\to 4}$ has however no good infrared properties, not even if restricted to the overall color singlet. Good IR-behavior is only obtained if one takes into account the complete set of diagrams that yield a state of four reggeized gluons in the t-channel, Fig. 5. The four reggeized gluon state coupling directly to the quark can be shown to yield (in the overall color singlet) further reggeization in the symmetric color sector plus a term which takes the form of another two-to-four transition:



(9)

Combining this extra term with the two-to-four reggeized gluon vertex $U_{2\to 4}$, one obtains the vertex $V_{2\to 4}$:



(10)

This vertex $V_{2\to 4}$ is well known from the study of the triple discontinuity of a six-point amplitude in [5] and is in particular infrared finite. Projecting pairs of reggeized gluons on the color singlet, $V_{2\to 4}$ yields then the triple Pomeron vertex.

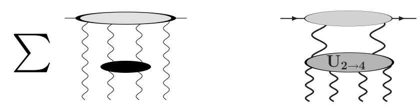


Figure 5. The state of four reggeized gluons in the t-channel may either arise from four reggeized gluons which couple directly to the quark and then interacted pairwise by the BFKL-Kernel and from a two-to-four transition.

In this contribution we presented a set of rules which allows to carry out longitudinal integration in the effective action, Eq. (1). These rules have been successfully applied to the derivation of reggeized gluon transition vertices from the effective action within the LLA and the obtained expressions are in agreement with earlier results. Future research should concentrate on extending these rules beyond the LLA.

[2]For explicit results we refer the reader to [4]

REFERENCES

1. L. N. Lipatov, Nucl. Phys. B **452** (1995) 369 [arXiv:hep-ph/9502308]; L. N. Lipatov, Phys. Rept. **286** (1997) 131 [arXiv:hep-ph/9610276],
2. M. Hentschinski, J. Bartels and L. N. Lipatov, arXiv:0809.4146 [hep-ph], M. Hentschinski, Acta Phys. Polon. B **39**, 2567 (2008) [arXiv:0808.3082 [hep-ph]].
3. V. N. Gribov, Sov. Phys. JETP **26**, 414 (1968) [Zh. Eksp. Teor. Fiz. **53**, 654 (1967)].
4. M. Hentschinski, "The high energy behavior of QCD: The effective action and the triple-Pomeron-vertex", (PhD-thesis) arXiv:0908.2576 [hep-ph].
5. J. Bartels and M. Wusthoff, Z. Phys. C **66** (1995) 157.